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An Approach for the Design Verification of Large Scale Safety Systems Based on the Use of Laguerre Polynomials Expansions.

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ABSTRACT: We present an application of Laguerre polynomial expansions as part of a set of techniques for the analysis and simulation of the dynamic response of Large Scale Systems (LSS), as in particular Nuclear Power reactors under transients and accidents. It is based on combining properties of piecewise linear systems (whenever large subsystems may be so approximated) with classical nonlinear time treatments of the rest.
The expansions provide an alternate numerical method for the inversion of Laplace transforms and allow an efficient coupling between frequency domain (linear subsystems) and time domain (non linear subsystems) approaches. It is illustrated through the application of the algorithm to Nuclear Power Plant models as the ones used in the analysis of stability in safety assessments.
The algorithm is key to an alternate and general numerical treament for modeling LSS that is particularly useful in a Protection Engineering context (i.e., application of reliability techniques to the design of safety systems). Indeed, piecewise linear systems are a natural way of describing consequences of sequences of events associated to protection actions, sequences that are a basic concept in the frame of PSA (Probabilistic Safety Assessment). As such is part of a series of papers devoted to a better description of the influence of the time evolution of process variables in the widespread ET/FT (Event Tree/ Fault Tree) PSA methodology, used in the regulations of several industries to protect installations, public and environment.
Keywords: Protection Engineering, System Safety, Damage Indicators, Sequence of Transitions, Probabilistic Safety Assessment, Theory of Stimulated Dynamics, Laguerre Polynomials Application, Large Scale Systems, Piecewise Linear Systems

I. INTRODUCTION. SCOPE AND OBJECTIVES.

In references [1] and [2] we described the potential benefits of using piecewise linear models in a Protection Engineering context (i.e., application of reliability techniques to the design of safety systems), as they are a natural way of describing consequences of sequences of events associated to protection actions, sequences that are a basic concept in Probabilistic Safety Assessment (PSA) techniques used in the regulations of several industries to protect installations, public and environment.

Most of those models involve large scale models. The simplest of them are Large Scale Linear System (LSLS) models because there is the possibility of projecting the influence of the whole system on a single variable $x_j(t)$ of the state vector $\overline{x}$, i.e., the time evolution of $x_j(t)$ may be found via transfer functions, i.e.

$$x_j(t) = \pounds^{-1}_{s\to t}\sum_{i=1}^{M} G_{j\leftarrow i}(s)\tilde{u}_i(s)$$
$$G_{j\leftarrow i}(s) \equiv \sum_{k=1}^{N}\left[s[I]-[A]\right]^{-1}_{j,k}[B]_{k,i} \qquad (1)$$

where $\pounds^{-1}_{s\to t}$ are the inverse Laplace operators, while matrices A and B describe the state variable formulation. While the dimension N of A is expected large, the number of inputs $u_i(s),\ i=1,2,...M$ is not expected so, provided theirs appropriate choice. Note that the state variables description is a large set of differential equations, while the transfer function approach that project into a single variable is an integral representation constituted by an additive set of M convolutions with the inputs, given by the combinations of the matrix elements of A and B involved in the definition of the transfer functions (1). When the information of interest is vehicled by only a few process variables (as it is the case in Protection Engineering see [1] and [2]), it is obvious the advantage of using projections via transfer functions, as the order of the system is embedded in (1), then becoming less relevant, and the number of transfer functions to handle becomes affordable.

Additionally, all the topological properties associated to signal flow processing techniques may be applied, including the possibility of collapsing large topologies or decomposing them (see examples below in Section 5), as well as finding envelopes of the time evolutions.

The great problem is that using time independent matrices A, B (thus, a single j,i set matrix of transfer functions) to model the system falls too short and is a serious limitation in this modeling approach. On the other hand, if A and B are assumed time dependent, the scope of application is excellent and many non-linear systems can be approximately modeled. An intermediate approach is of course to consider a finite set of matrices A, B spanning the time interval, i.e., the piecewise linear approach.

Also in references [1] and [2] we defined transmission functions as extensions to piecewise linear systems of transfer functions that apply within each finite time interval. It is straightforward to describe them in terms of products of transfer function matrices of different Laplace variables associated to the different pieces. However, the number of transfer functions involved is so large that the approach cannot be used to compute them, only to define them.

As shown in reference [2], we properly call transmission function theory (TFT) a specific alternate representation that makes the computation of transmission functions tractable. To understand it, in that reference we show how to obtain the solution in terms of the case B=I and we may rewrite (1) for that case as

$$G_{j\leftarrow i}(s) \equiv \left[s[I]-[A]\right]^{-1}_{j,i} = \frac{\sum_{k=0}^{N-1} Q^{k}_{j,i} s^{k}}{P(s)} \qquad (2)$$

with $P(s) \equiv \det\left[sI-[A]\right]$ the characteristic polynomial, $[A]$ being a NxN matrix. This equation shows that the three indices of Q are related between themselves and one of them is redundant, which is the basic reason for the intractability of the matrix product formulation. A main feature of the alternate representation is that it will only involve the set of matrices G with fixed j, avoiding the main problem of the products.

TFT also shows that all the nice properties for large scale single piece systems are then extended to several intervals as well, which justifies its adoption and the need to look for TFT numerical methods that benefit from this. The rest of this paper will then concentrate on adequate numerical methods to handle LSLS such that they may be extended to LSS where large subsystems can be piece-wise linearized (LSPS).

More specifically, the purpose of the paper is to present an application of Laguerre polynomial expansions as basic technique for the analysis and simulation of LSLS dynamic response. An additional paper will afford the more complex case of their optimal extension to LSPS.

The paper presents a feasible strategy to obtain the collapsed description and time domain simulation of a block diagram of linear systems. Section 2 states the simulation problem, while Section 3 describes the automatic implementation of collapsing techniques. Section 4 introduces the orthogonal expansion functional basis (Section 4.1), and several methods to obtain its coefficients (Section 4.2). It also shows its usefulness for model identification (Section 4.3) and formally discusses the convergence and accuracy of the resultant Laguerre series (Section 4.4). Section 4.5 presents the numerical algorithm for the convolution of the expansions with the input signals (Laguerre convolution algorithm). Section 5 illustrates the techniques with two significant examples and Section 5.1 serves the purpose of illustrating its use as a model identification technique. Conclusions are given in Section 6 and details in appendices A, B, C.

II. PROBLEM DESCRIPTION

For LSS models with a structure of LSLS sections coupled to non-linear sections, neither frequency nor time domain analysis techniques are fully satisfactory. Frequency domain analyses do not drive easily into calculation of the system transient behavior and do not identify possible non-linear instabilities leading to limit cycles or the like [3]. On the other hand, the use of more realistic non-linear models, hardly allows for systematic parametric studies to obtain a significant picture of the system behavior under a wide spectrum of conditions. Both, computer and user time consuming trial approaches, with too many parameters to examine, are often necessary.

The classical Laplace frequency domain has been almost abandoned as a useful simulation tool, in spite of its recognized power, particularly for stability analysis [4] and distributed parameter linear systems [5], as well as its potential for reducing the large and complex linear sections to a set of collective transfer functions, linear feedback free and able to be coupled to the time domain nonlinear parts. Moreover, the computer time involved in the inversion to the time domain of the transfer functions at each time step and the difficulty of its coupling to the non linear elements have also contributed to its seldom use today.

Alternate techniques, like time domain matrix solution of first order differential equation descriptions or use of discretization and the z-transform, are common. However, these methods also have some disadvantages:

a) For large linear sections, the matrix formulation involves large matrix operations whose size increase with the system complexity. Moreover, many of these techniques require a full identification of the eigenvalues (i.e., poles) of the system. Similarly, the set of difference equations that constitute the z-transform solution for lumped systems becomes dependent on the system size, and of the sampling period ([6]).

b) Both techniques cannot handle distributed parameter linear systems, represented by non rational transfer functions, in a simple and general way. In fact, one nice feature of our approach is that it can be used to automatically obtain an optimized lumped parameter representation of the system in this case.

The easy coupling between the frequency and time domains has been made possible due to the development of the simulation method, described in this paper, that allows to easily convert the collective (collapsed) transfer functions Laplace domain into the time-domain. This collapsed transfer function, or its equivalent convolution kernel, carries out all the essential information of the system description.

As early as 1966, R. Bellman has shown ([7]) the great potential of integral transforms and orthogonal polynomials for the numerical computation of linear and non linear differential equations, in particular for reproductive systems. In the sections that follow, we present an approach that alleviates some of the difficulties of the classical theory, by working in the time domain with the step response of the Laplace transfer functions, while maintaining most of the nice properties of the discrete formulations. The major observation leading to it is that the convolution property of linear systems is its essential feature, and all the information is carried out by the convolution kernel. Once this kernel is found, there is no reason for increasing the complexity of the solution with the system size.

So, the problem to be solved is to efficiently compute the convolution integral:

$$o(t) = \int_0^t k(t-\tau)i(\tau)d\tau \qquad (3)$$

that describes the output $o(t)$, from an initial steady state, of a linear system with impulse response $k(t)$ (i.e., with transfer function $K(s)$ Laplace transform of $k(t)$, when the input signal is $i(t)$.

Some intrinsic difficulties of problem defined by Eq. (3) are the following:

a) Generally, it is easier to represent a linear system by its transfer function $K(s)$, than by its convolution kernel $k(t)$. Although many techniques have been devised for the inversion to the time domain, they have drawbacks when facing the rest of the requirements.

b) In a simulation process, the input signal $i(t)$ is known while the process is running, i.e., for the time interval $(0,t)$, but not its future. This prevents the use of any kind of conventional techniques for inversion of $K(s)I(s)$.

c) The "non cumulative" character of the convolution integral, (i.e., the function to be integrated at time t as well as the integration range, change also with t), complicates conventional techniques of numerical integration. Moreover, it is very convenient to be able to advance in time with only the information of a few previous time steps. This is only easy for exponentials, but reducing the kernels to exponential combinations requires identification of the eigenvalues (poles).

Laguerre polynomials having simple convolution properties and being a complete orthogonal basis in the interval $[0,\infty)$ (i.e., the variation range of the time independent variable) become then a natural tool. Additional features like its easy integration and close connection with the Laplace and z transformations enhance their value for the treatment of large linear systems, particularly for the distributed parameter case where other techniques can not be easily applied. In fact, considerations i), ii), iii) above indicate that the expansions in an appropriate Laguerre basis are almost unique to face these difficulties. Other interesting features have been discussed by several authors (see for instance [8], [9]).

The basic idea is then to express convolution kernels of linear systems in the form of expansions in functions based on these polynomials. Of course, Laguerre expansions are not panacea, and a careful selection of the parameters controlling them is required, (see Section 4.4 and Ref. [10]) to ensure reasonable results with a finite number of the expansion terms.

III. COLLAPSING LINEAR BLOCK DIAGRAMS.

A way to improve the efficiency of the simulation of LSS large networks, lumped or distributed (i.e., rational or non-rational transfer functions), is the reduction of the system complexity (in terms of number of interconnected elements and feedback loops) by using classical signal flow processing topological techniques for the LSLS subsystems which are then described in terms of a relatively reduced set of parameters. This task is afforded by applying Mason's rule ([11]) and/or their succesors ([12, 13] to any input-output linear system pair reducing them to a set of colective transfer functions, feedback free and able to be coupled to time domain, non linear simulators.

The functionality required by this approach has been implemented in the pre-processor code COLAPSO [14] that constitutes a module of the computational platform SCAIS (Simulation Code System for Integrated

Safety Assessment) developed by CSN ([15]). Emphasis has been placed in a modular structure, quite similar to simulation languages, allowing for a fast implementation of changes in the system configuration and/or settings.

COLAPSO main capabilities are the following:

1. Automatic reduction of a linear system to a minimum set of transfer functions.
2. Stability tests and/or spectral analyses.
3. Generation of the Laguerre coefficients to be used by SCAIS for each of the collapsed transfer functions.
4. Automatic generation of a new input data set equivalent to the original input, with all the linear parts reduced to its minimum configuration and described via an optimum Laguerre expansion.

COLAPSO identifies linear and non-linear parts and all the inputs and outputs of the linear parts within a block diagram topology. By using an automatic and modernized version of Mason's rule, it identifies all the paths and loops in the linear sections building-up a transfer function for every input-output pair.

This approach is then suitable for systematic parametric studies on the influence of physical parameters on stability and time response. For stability and spectral analysis COLAPSO, compute the Mason's determinant, determines the Nyquist/Bode plots as a generalization of the open loop concept of single loop feedbacks and applies Nyquist criterion checks for stability. Parametric stability may be studied via characteristics derived from the Nyquist plot (minimum distance to the critical point, gain margin, phase margin, ...)

For simulation purposes, COLAPSO calculates as many Laguerre expansion coefficients (see Section 4.2) as specified for each final collapsed transfer function. The fewer Laguerre coefficients used to describe the system, the more efficient will be their use as intermediate lumped parameters. In addition, spectral analysis helps eliminate linear input/output pairs if shown unimportant, and to optimize the truncation of the Laguerre series. In this way the frequency analysis helps the simulation efficiency.

From the topological stand-point, COLAPSO parallels similar developments (see [12] [13]), but adding the possibility of treating distributed linear systems i.e., represented by non-rational transfer functions (see Section 5 below).

IV. LAGUERRE ALGORITHM FOR LINEAR SYSTEMS.

4.1 General.

A transfer function $K(s)$, describing a linear system with impulse response $k(t)$, can be expanded as follows [16]:

$$K(s) = \sum_{n=0}^{\infty} k_n^{\lambda\mu} \left(1 - \frac{\lambda}{s+\mu}\right)^n = \\ = \pounds\left(\sum_{n=0}^{\infty} k_n^{\lambda\mu} e^{-\mu t} \frac{d}{dt}\left(L_n(\lambda t)\theta(t)\right)\right) \tag{4}$$

In this equation $\pounds$ is the symbol of the Laplace transform, $\{L_n(t)\}$ are the Laguerre polynomials; μ is a real parameter less than but close to the real part of the dominant pole of the system; λ is a time scale real parameter; $\theta(t)$ is the step function; $k_n^{\lambda\mu}$ are here called "$\lambda\mu$-Laguerre coefficients" of the system $K(s)$ and the remainder of the term under the second summation sign are the "Laguerre functions" $e^{-\mu t}\frac{d}{dt}\left(L_n(\lambda t)\theta(t)\right)$ on which the expansion is based.

This particular expansion of the transfer function is supported by the following:

1. The functions $\left\{\frac{d}{dt}\left(L_n(t)\right)\right\}$ constitute a complete orthogonal basis of the space $L^2(0,\infty)$ with weight te^{-t}. Thus, any function $k(t)$ of that space can be expanded in these polynomials, for real nonnegative values of t.
2. The truncation of a Laguerre expansion of type (4) with N terms, denoted as $k_N(t)$, minimizes the following error function:

$$\delta = \int_0^{\infty} \left|k(x/\lambda) - k_N(x/\lambda)\right|^2 e^{((2\mu/\lambda)-1)x} dx \tag{5}$$

This avoids the risk of error amplification when taking the inverse Laplace transform.

3. The convolution of two Laguerre functions with the same λ and μ, is another Laguerre function with the same λ and μ. This implies that the coefficients of

$$K(s) = K^1(s)K^2(s) \tag{6}$$

satisfy the following convenient relationship

$$k_n^{\lambda\mu} = \sum_{j=0}^{n} \left(k^1\right)_j^{\lambda\mu} \left(k^2\right)_{n-j}^{\lambda\mu} \tag{7}$$

that is, the discrete convolution among the λ,μ -Laguerre coefficients of both systems.

4.2 Computing Laguerre coefficients

The selected Laguerre basis being an orthogonal set, the coefficients may be obtained via Parseval´s theorem. For any kind of linear system, i.e., both rational and non-rational transfer functions, a general way is ([16]):

$$k_n^{\lambda\mu} = \frac{1}{2\pi j}\int_{\Gamma} \frac{K(\lambda s-\mu)}{s}\left(\frac{s}{s-1}\right)^n \frac{ds}{1-s} \tag{8}$$

The integration path Γ is a closed contour around the point $(1,0)$ in the s-plane, containing no singularities of $\frac{K(\lambda s-\mu)}{s}$. Other procedures have been identified, that apply with advantage in some cases, for instance when $K(s)$ is a rational function of s (i.e., lumped parameter systems see Appendix 1) or if the information about $K(s)$ is given in the $K(jw)$ frequency range. Other possibility is shown in next section.

4.3 Model identification techniques.

In addition, $\lambda\mu$ -Laguerre coefficients satisfy the following set of recurrences:

$$\begin{aligned} k_n^{\lambda\mu} &= \lim_{s\to\lambda-\mu}\left(s+\mu-\lambda\right)X_n^{\lambda\mu}(s) \\ X_n^{\lambda\mu}(s) &= \left(\frac{s+\mu}{s+\mu-\lambda}\right)\left[X_{n-1}^{\lambda\mu}(s)-k_{n-1}^{\lambda\mu}\right] \\ X_0^{\lambda\mu}(s) &= \left(\frac{1}{s+\mu-\lambda}\right)K(s) \end{aligned} \tag{9}$$

which provide a model identification method of the system according to (4), when the impulsive response of the system $k(t)$ is known experimentally. Figure 1 presents and sketchs these recurrences, and introduces an identification method for a Laguerre expansion model of the system, i.e., parametric, off-line, temporal and recursive identification method. The well-known final value theorem for linear systems is at the basis of the proposal:

$$\begin{aligned} k_n^{\lambda\mu} &= \lim_{u\to 0} u X_n^{\lambda\mu}\left(u+\lambda-\mu\right) = \lim_{u\to 0} u\pounds\left[e^{-(\lambda-\mu)t}x_n^{\lambda\mu}\left(t\right)\right] = \\ &= \lim_{t\to\infty} e^{-(\lambda-\mu)t}x_n^{\lambda\mu}\left(t\right) = \lim_{t\to\infty} k_n^{\lambda\mu}\left(t\right) \end{aligned} \tag{10}$$

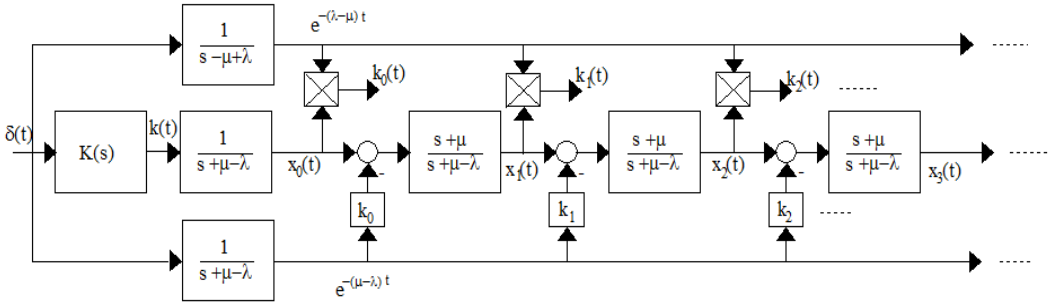


Figure 1. Filter ladder diagram to obtain the $\lambda\mu$ -Laguerre coefficients of a system with impulsive response $k(t)$.

If K(s) is known, the $\lambda\mu$-Laguerre system coefficients may then be obtained as the final value of the $k_n^{\lambda\mu}(t)$ functions defined as in the last relations (10). When the information is in the form of time dependent input-output pairs, for instance as a result of experiments, the $\lambda\mu$-Laguerre coefficients of the system would be obtained by deconvolving via relationship (7) the λ,μ-Laguerre coefficients of $o(t)$ and $i(t)$, identified with (10) by considering the convolution kernel of:

$$k_n^{\lambda\mu} = \frac{1}{i_0^{\lambda\mu}}\left[o_n^{\lambda\mu} - \sum_{m=1}^{n} k_{n-m}^{\lambda\mu} i_m^{\lambda\mu} \right] \tag{11}$$

4.4 Choice of parameters

Parameters λ and μ are key to the method, their particular choice changing significantly the characteristics of the Laguerre series. In fact the potential instability of $k(t)$ i.e.,

$$\int_0^{\infty} |k(t)|\,dt \qquad \text{is unbounded} \tag{12}$$

is reflected in the lack of convergence of the Laguerre series. When μ is taken higher than the dominant pole of $K(s)$, and or $\lambda > 2\mu$ the series does not converge. On the other hand, with μ below but close to the real part of the dominant pole and high λ the convergence is faster, so less coefficients are necessary to describe the system.

It is of special interest the case $\lambda = \mu$, in which the first coefficient becomes the system static gain, describing the stationary behavior, the truncated series being a good solution for $t \geq t^*$ (the larger N the smaller t^*). This is an important feature that helps the treatment of stiff systems (i.e., widely different eigenvalues) and allows asymptotic calculations with a few coefficients.

4.5 Laguerre algorithm.

The response $o(t)$ of the system to an input $i(t)$ in Eq. (3), can be then obtained as:

$$O(s) = \sum_{n=0}^{\infty} k_n^{\lambda\mu} \left(1 - \frac{\lambda}{s+\mu}\right)^n I(s) \equiv \sum_{n=0}^{\infty} k_n^{\lambda\mu} \psi_n(s) \tag{13}$$

where the functions $\psi_n(s)$ are only dependent on λ, μ and the input signal $i(t)$, while $\left\{k_n^{\lambda\mu}\right\}$ depends on the system. In the time domain, these functions satisfy the generic set of ordinary differential equations:

$$\left(\dot{\psi}_n(t) - \dot{\psi}_{n-1}(t)\right) + \mu\left(\psi_n(t) - \psi_{n-1}(t)\right) = -\lambda\psi_{n-1}(t) \tag{14}$$

with boundary conditions:

$$\begin{aligned} &\psi_0(t) = i(t) \quad n = 0; \forall t \\ &\psi_n(0) = 0 \qquad n \neq 0 \end{aligned} \tag{15}$$

Eq. (14) and (15) may be solved by i) conventional differential equation techniques, ii) z-transform, iii) convolution integrals. The one that has been chosen as effective and coherent with the initial problem defined by Eq. (3), is to consider them as the set of nested universal convolutions:

$$\begin{aligned} &\psi_n(t) = \psi_{n-1}(t) - \lambda\int_0^t e^{-\mu(t-\tau)}\psi_{n-1}(\tau)d\tau \\ &\psi_0(t) = i(t) \end{aligned} \tag{16}$$

Much care must be taken in solving this set of equations, because the nested convolutions can amplify the error effects of the numeric sampling of $i(t)$. The following has been obtained when the input signal can be written as a linear function within the interval (t_k, t_{k-1}):

$$\psi_n(t_k) = \sum_{m=0}^{n} \alpha_{n-m}\left[\psi_m(t_{k-1}) - \psi_0(t_{k-1})\right] + \\ + i(t_k)\phi_n(\lambda\Delta t) + \left(i(t_k) - i(t_{k-1})\right)\frac{1}{\lambda\Delta t}\int_0^{\lambda\Delta t}\phi_n(z)dz \tag{17}$$

where $i(t_k)$ and $i(t_{k-1})$ are the values of the input signal at the current and previous time steps, and α_n and ϕ_n are the following universal functions of μ/λ and $\lambda\Delta t$:

$$\alpha_n \equiv e^{-\mu\Delta t}\left(L_{n+1}(\lambda\Delta t) - L_n(\lambda\Delta t)\right) \tag{18}$$

$$\phi_n(x) \equiv e^{-(\mu/\lambda)x}L_n(x) + \frac{\mu}{\lambda}\int_0^x e^{-(\mu/\lambda)y}L_n(y)dy = \\ = \int_0^x e^{-(\mu/\lambda)y}\frac{d}{dy}L_n(y)dy \tag{19}$$

These paramerical functions satisfy several recursion formulae useful for their computation (Appendix B).

The Laplace inversion of (13) taking into account (17) gives:

$$o(t_k) = \sum_{n=0}^{N} v_n\psi_n(t_{k-1}) - Bi(t_{k-1}) + (A+B)i(t_k) \tag{20}$$

where v_n, A and B, are given by:

$$v_n = \sum_{m=n}^{N} k_n^{\lambda\mu}\alpha_{m-n} \tag{21}$$

$$A \equiv \sum_{n=0}^{N} k_n^{\lambda\mu}\phi_n(\lambda\Delta t) = o_{step}(\Delta t) \tag{22}$$

$$B \equiv \sum_{n=0}^{N} k_n^{\lambda\mu}\frac{1}{\lambda\Delta t}\int_0^{\lambda\Delta t}\phi_n(z)dz = \overline{o}_{ramp}(\Delta t) \tag{23}$$

and do not vary from one time step to another (i.e., they can be pre-calculated). Note that equations (17) to (23), here called the Laguerre algorithm, also provide provide an efficient numerical method for inverting Laplace transforms of any function [19].

When the linear system is a part of a non-linear feedback loop (see for instance the reference system of Section 2), the simulation driver takes advantage of the form of (20). Since the two former terms in the right-hand side of (20) depend only on the system and the input signal up to the time t_{k-1}, they do not need to be recalculated (see paragraph iii/ of Section 2) at each new time step, saving iteration time and/or allowing for larger time steps.

It must be finally remarked that the algorithm allows a generalization, function of n previous time steps, if the input signal would be expressed as a polynomial of n-th order within each time step. Algorithm optimization relative to nth order and parameter selection as well as the stiffness treatment is important [17, 18]. Because our final goal is a LSPS numerical algorithm, this problem will be studied in the associated paper.

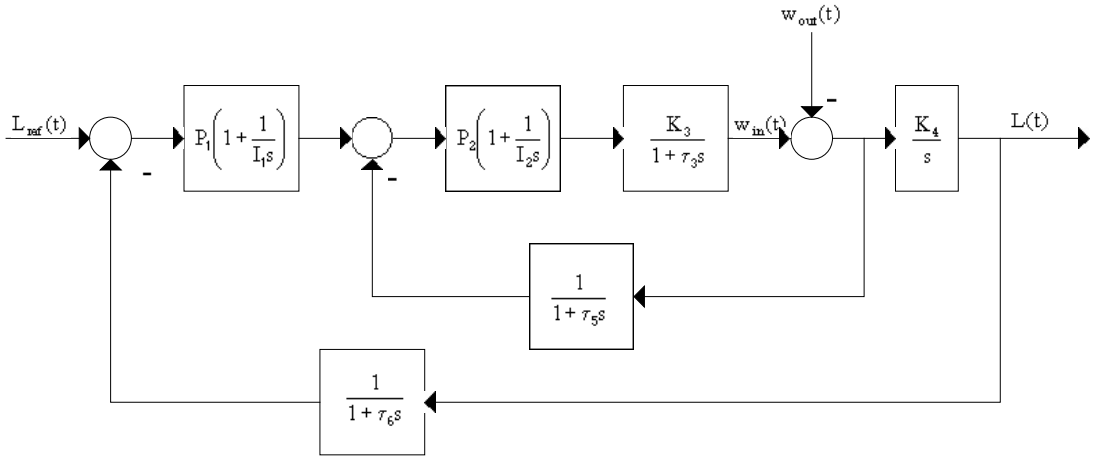
V. APPLICATIONS

For illustration of the method we have selected the analysis and simulation of a tank control level and the primary circuit of a pressurized water nuclear power plant (PWR).

Figure 2 shows the block diagrams of a typical control level of tank or steam generator on process plants with typical double feedaback loop on level and flow mistmatch, illustrating some reference example of topological complexity that the proposal of Laguerre method aimed to solve.

Frequency response of the collapsed system, $L(s)/L_{ref}(s)$ and $L(s)/W_{out}(s)$ (figure 3), is used to obtain an estimation of the dominant singularities. Values of $\mu \approx 0.09$ and $\mu \approx 0.0007$ have been estimated, and the case $\lambda = 2\mu$ is used for the sake of a faster convergence in the expansions, obtaining the following set of Laguerre coefficients:

$$\{-0.3622, -0.8515, -0.6469, -0.1907, -0.0313, 0.0123, \ 0.0229, \\ 0.0252, 0.0255, 0.0253, 0.0249, 0.0246, 0.0243, 0.0239, 0.0236, 0.0233,....\} \quad (24)$$



P1=0.0625; I1= 1000; P2=0.75; I2=1000; K3=1; T3= 5; K4= 0.25; T5=1; T6=1

Figure 2. Tank control level block diagram.

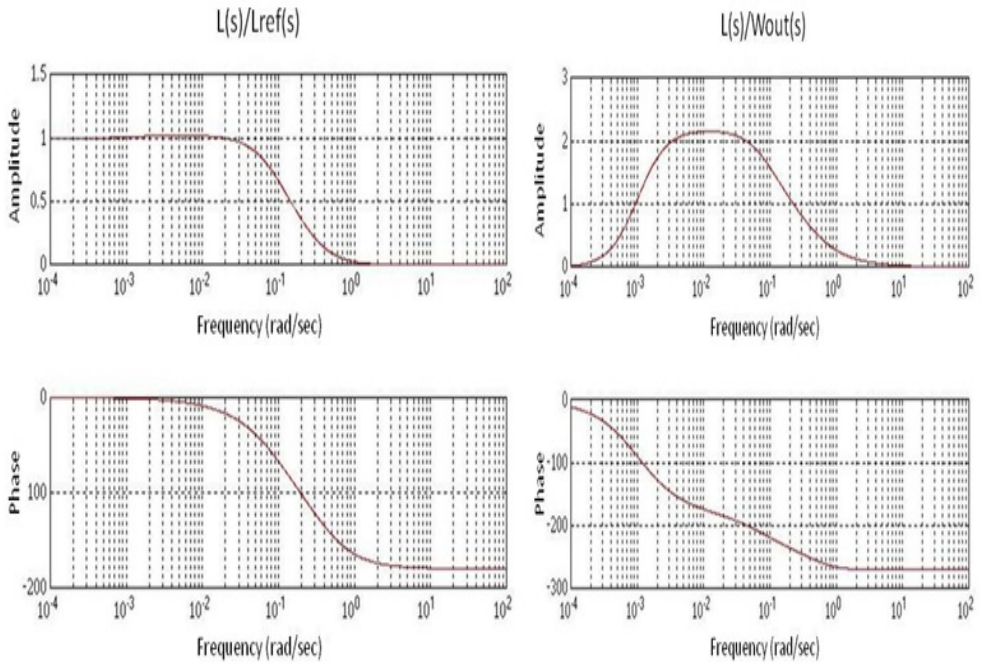


Figure 3. Frequency response of $L(s)/L_{ref}(s)$ and $L(s)/W_{out}(s)$ transfer functions.

Slow convergence of coefficients in (24) indicates the existence of stiff singularities compared with the transient scale represented by μ; decay value of coefficients $\rho = k_{n+1}/k_n$ allows to estimate its value M with:

$$M = \mu + \frac{\rho}{1-\rho} \approx 0.107 \quad (25)$$

Figures 4, 5 and 6 present the frequeny response and, the time response to step and sawtooth inputs, for the exact and 3rd and 5th Laguerre approximations. Slow decay to final values of step responses indicates the existence of the stiff pole.

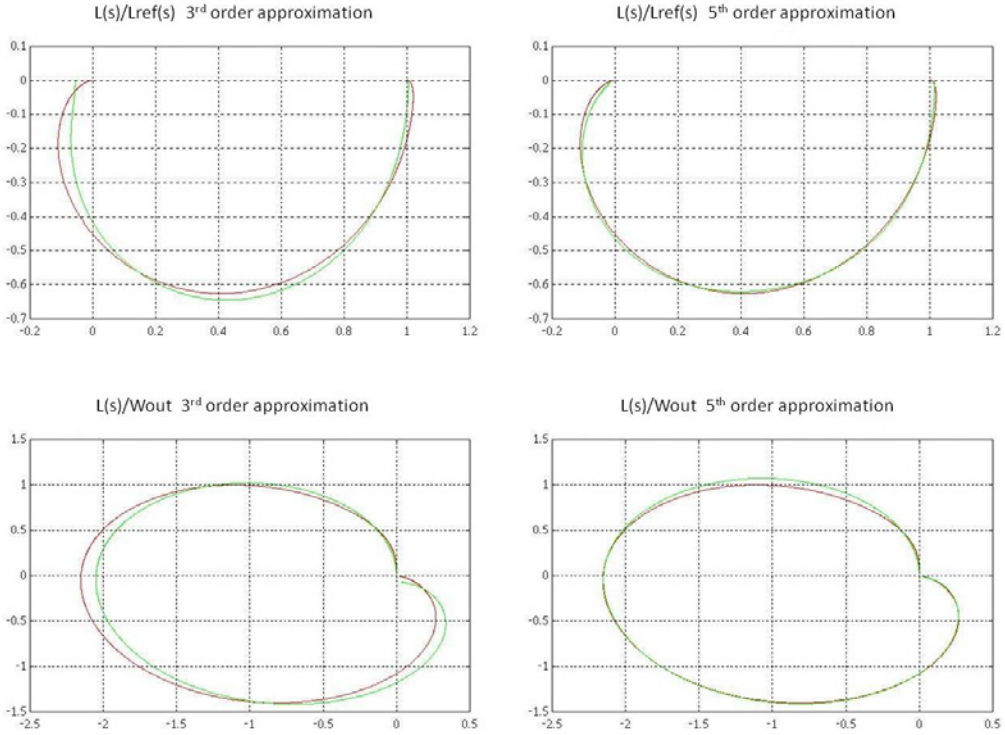


Figure 4. Exact and approximated (3rd and 5th order Laguerre expansion) Nyquist diagrams of $L(s)/L_{ref}(s)$ and $L(s)/W_{out}(s)$ transfer functions.

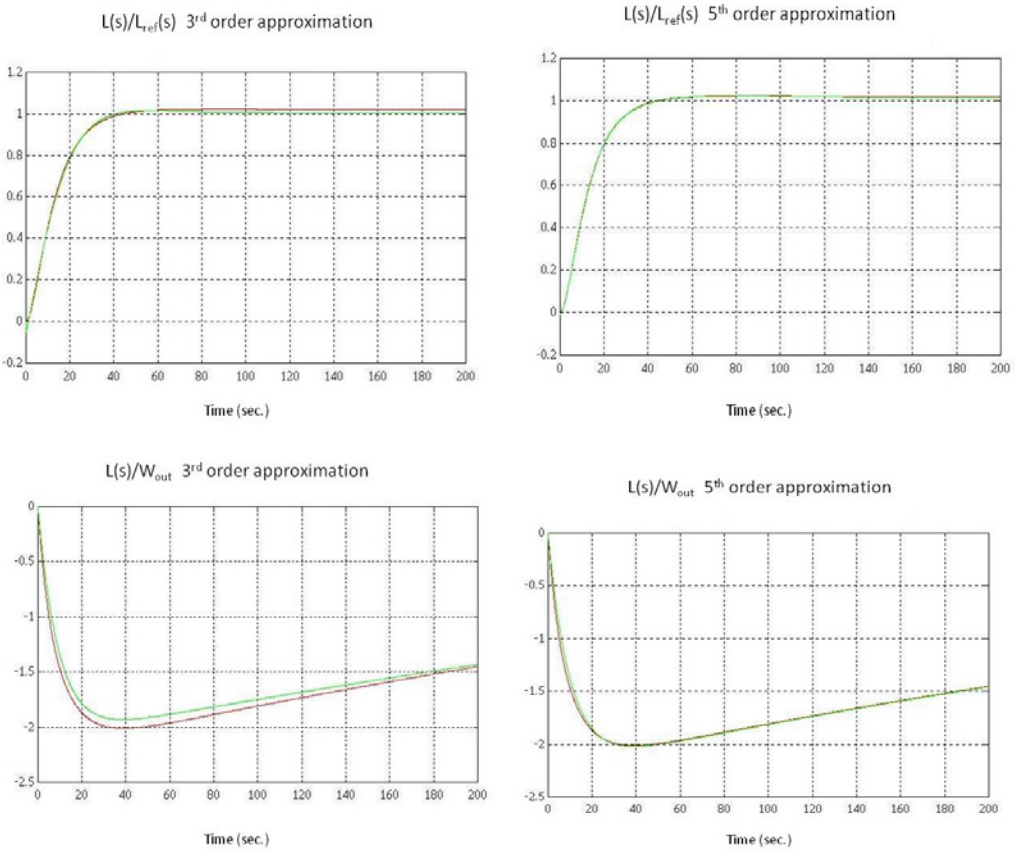


Figure 5. Exact and approximated (3rd and 5th order Laguerre expansion) step responses of $L(s)/L_{ref}(s)$ and $L(s)/W_{out}(s)$ transfer functions

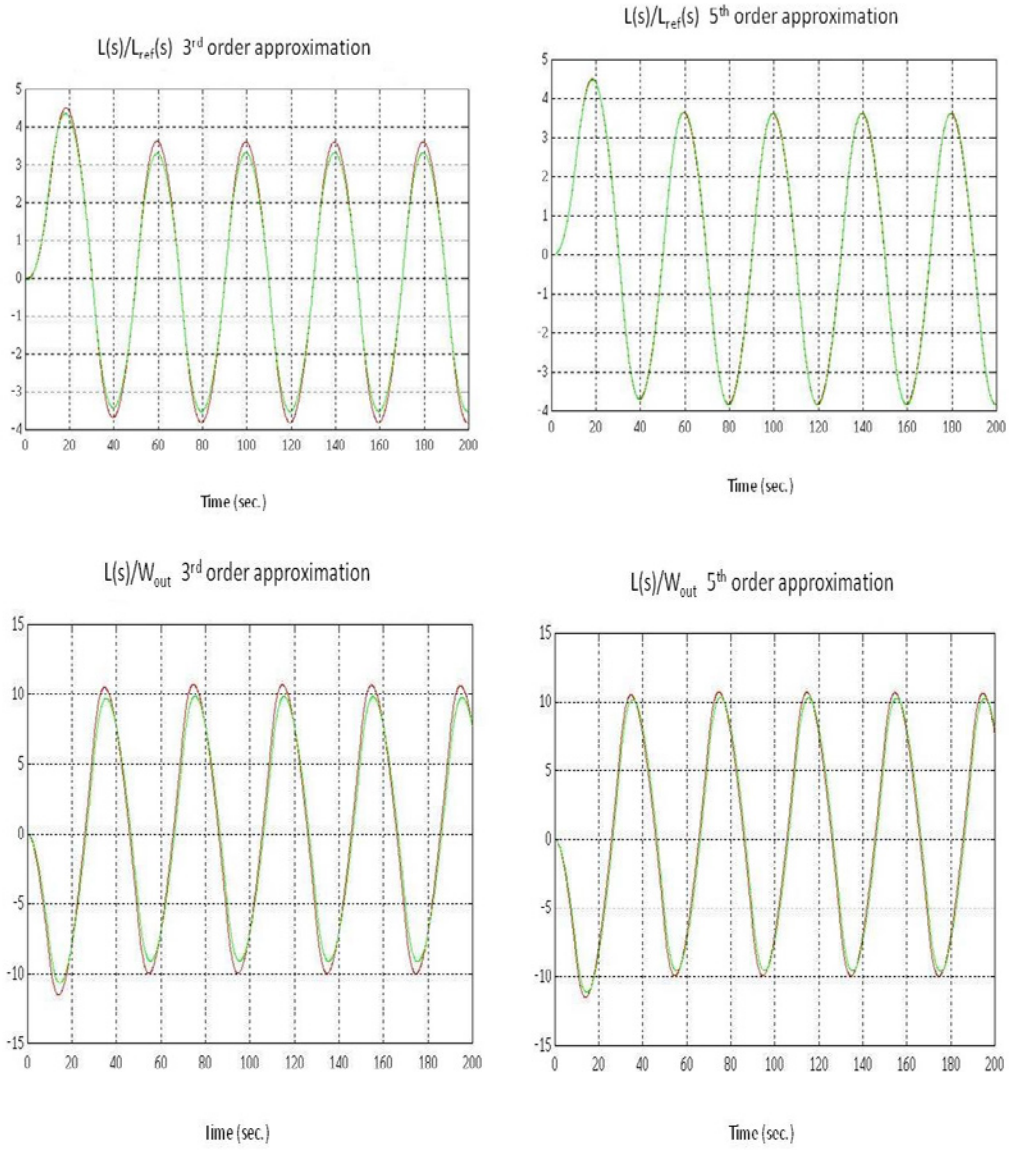


Figure 6. Exact and approximated (3rd and 5th order Laguerre expansion) sawtooth responses of $L(s)/L_{ref}(s)$ and $L(s)/W_{out}(s)$ transfer functions.

Figure 7 presents the block diagram of a linear model of the primary circuit model of a PWR, including the heat generation and transport processes from source to sink of the plant [20]. The reactor core, including lumped parameter neutronic and fuel thermal conduction aspects, enthalpy transport through heated and unheated pipes and steam generator primary and secondary sides are considered. The comprehensive list of transfer functions is given in Appendix 3. Note the complexity of the distributed parameter transfer functions used to describe the fluid transport, the pure non-linear effect of the reactor kinetics taken into account through the nonlinear multiplier type feedback (figure 7), and the topological complexity of the linear block diagram.

Nuclear power and secondary side temperature are used as external input signals (i.e., boundary conditions) to the model, whereas the core and steam generator heat flux, and the coolant hot, cold and average temperatures at both core and steam generator are used as output variables. A total of 20 linear blocks, with seven linear feedback loops can be identified in this block diagram to generate an equivalent system containing 12 transfer functions without linear feedback loops, can be identified in this block diagram.

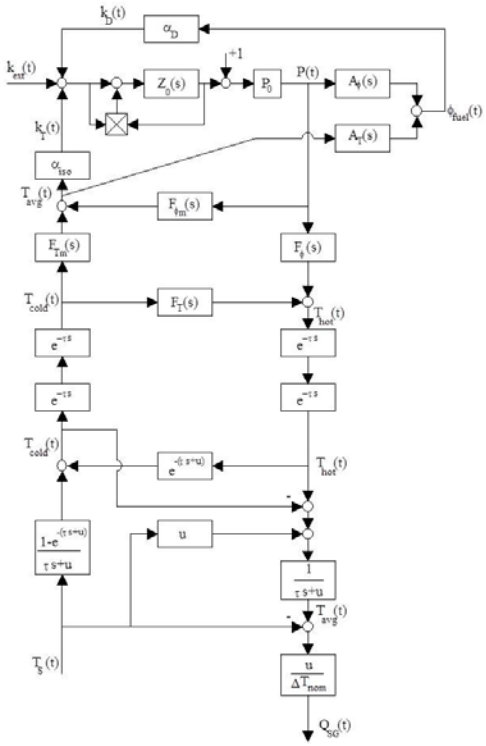
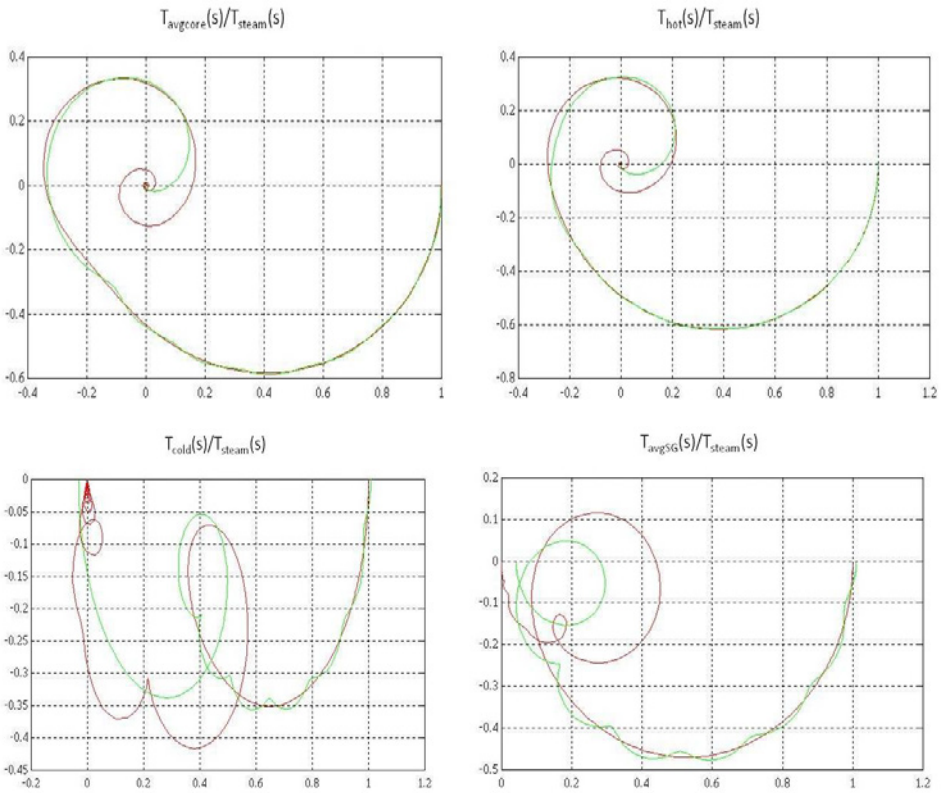


Figure 7. Nuclear Steam Supply System Block Diagram.

Frequency response (Bode diagrams) of this collapsed system is used to obtain an estimation of the dominant singularities used as values of parameter μ; the $\lambda = 2\mu$ is considered. Figures 8 and 9 show the good agreement of the exact and approximated (15th order Laguerre approximation) Nyquist diagrams.



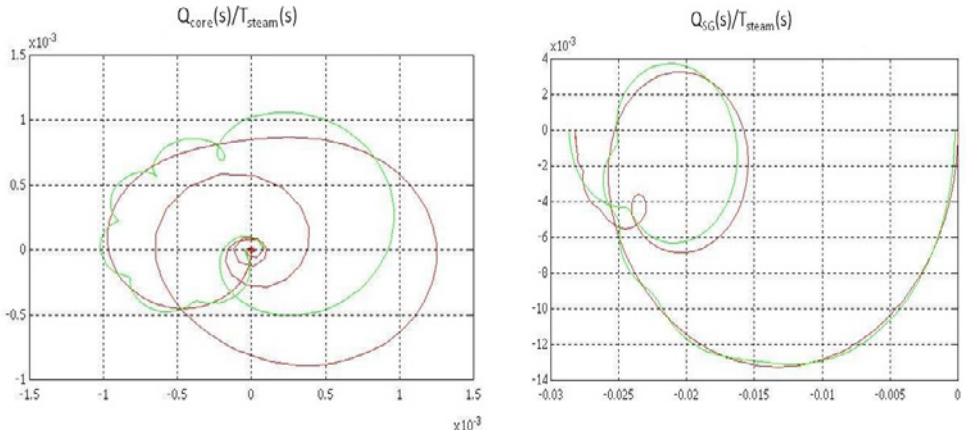


Figure 8. Exact and approximated (15th order Laguerre expansion) Nyquist diagrams of PWR collapsed transfer functions (corresponding to Steam Generator secondary side temperature).

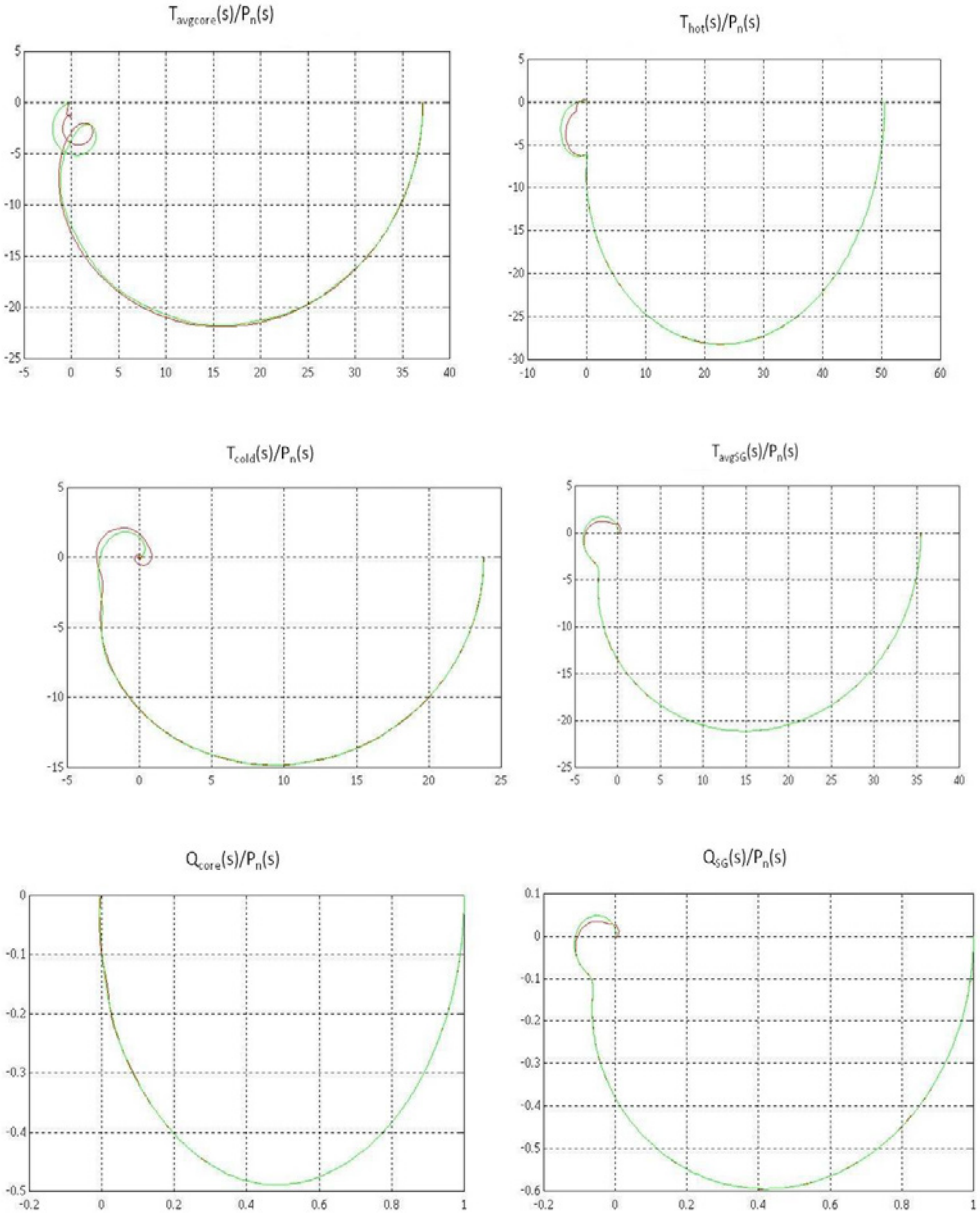
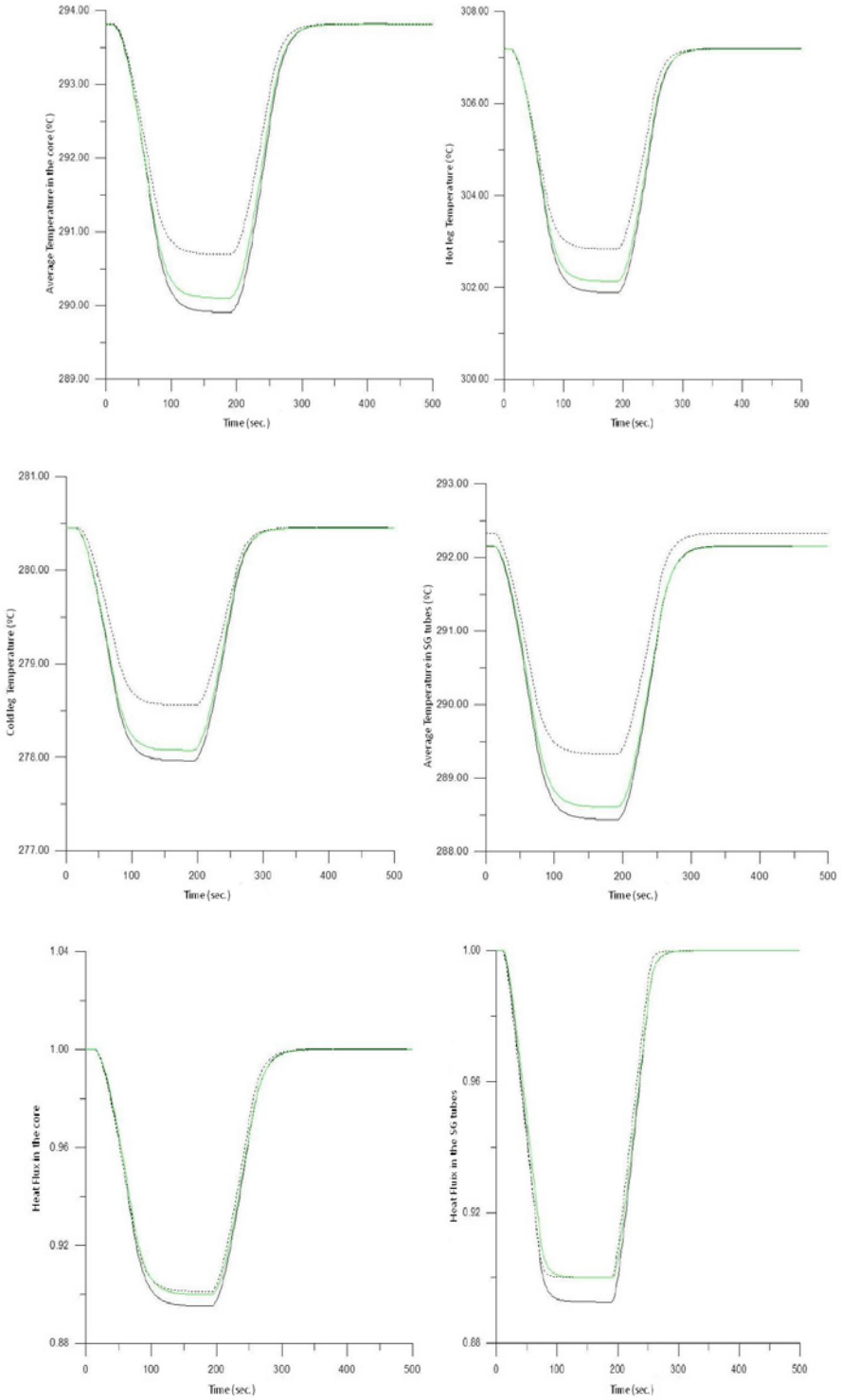
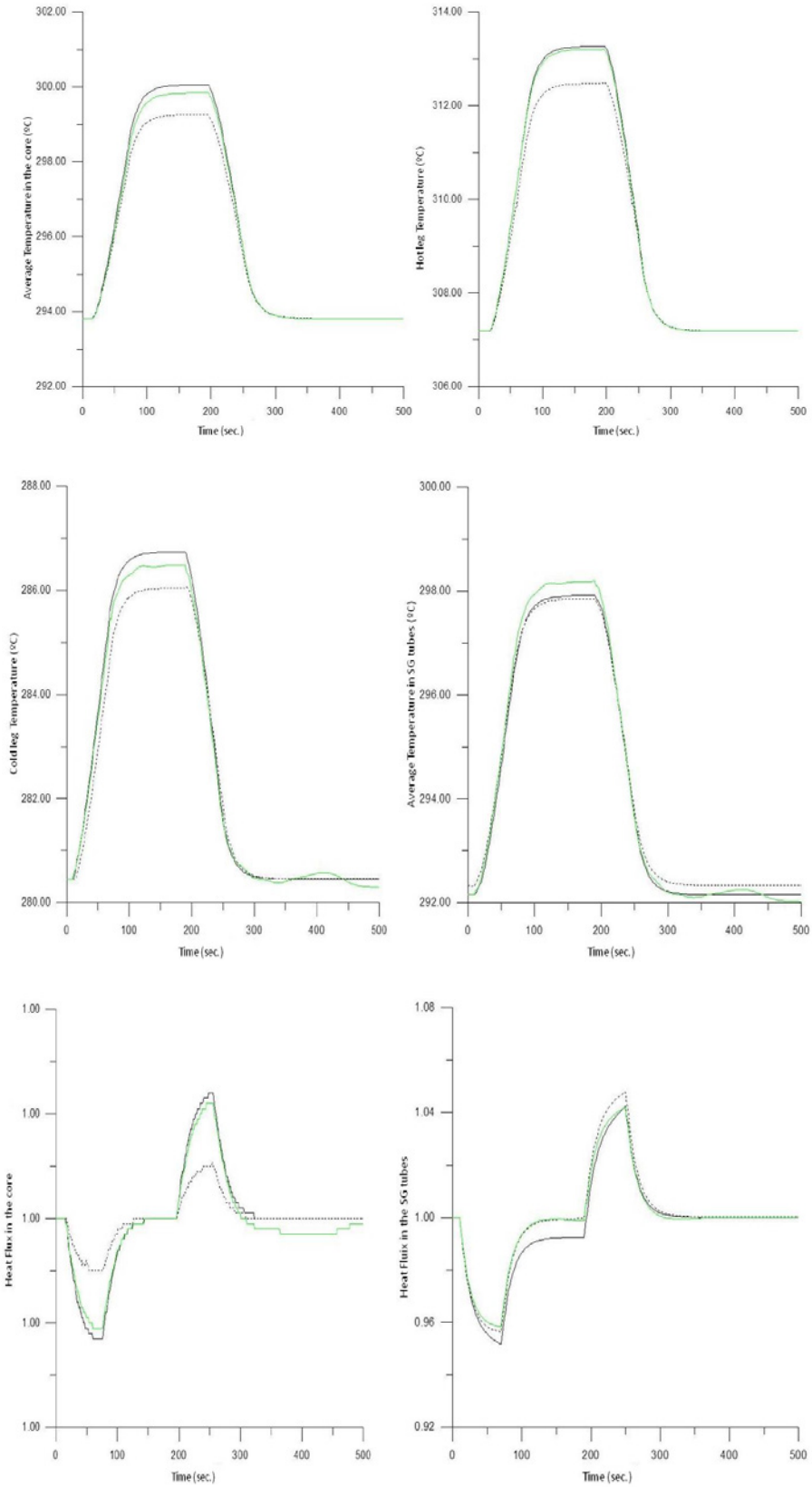


Figure 9. Exact and approximated (15th order Laguerre expansion) Nyquist diagrams of PWR collapsed transfer functions (corresponding to Core Power).

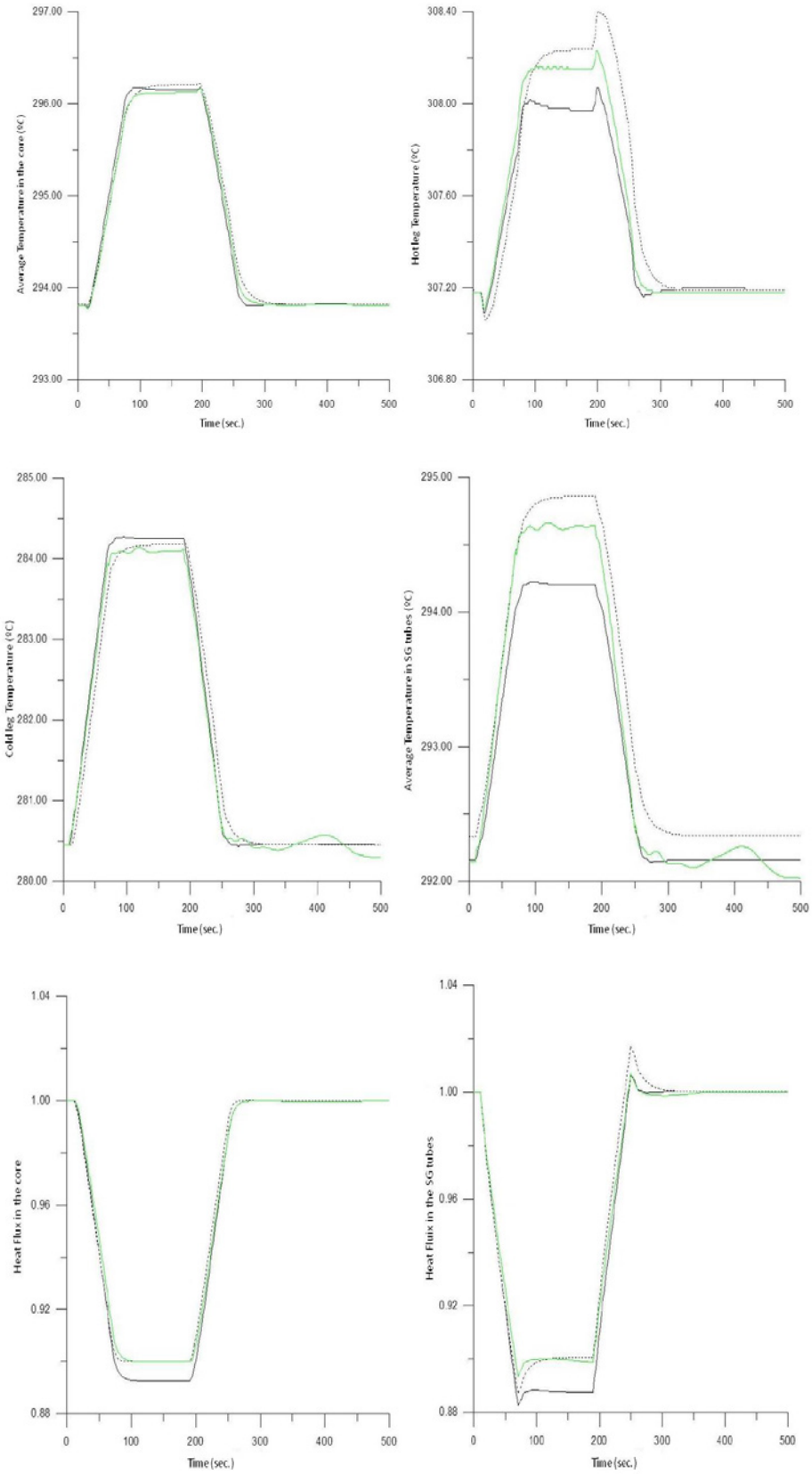
Time responses of the approximations of 15^{th} order with and without collapsing are used to simulate the dynamic behaviour of the process for different type of nuclear power and steam temperature input functions, and compared against the exact time response for the hot and cold legs, average temperature, and heat flows in the core and in the steam generator tubes are given in figures 10, 11 and 12. Figures 10 collect time responses after the sudden drop from 100% to 90% in the nuclear power, figure 11 the time responses after an increase of 6° C in the temperature of the secondary steam, and figure 12 time responses when these both perturbations occur simultaneously. Figures show the good agreement of the exact and approximated (15th order Laguerre approximation).



Figures 10. Exact and approximated (15th order Laguerre expansions) time responses of the core power perturbation.



Figures 11. Exact and approximated (15th order Laguerre expansions) time responses of the temperature of the steam perturbation.



Figures 12. Exact and approximated (15th order Laguerre expansions) time responses of the simultaneous perturbations in core power and temperature of the steam.

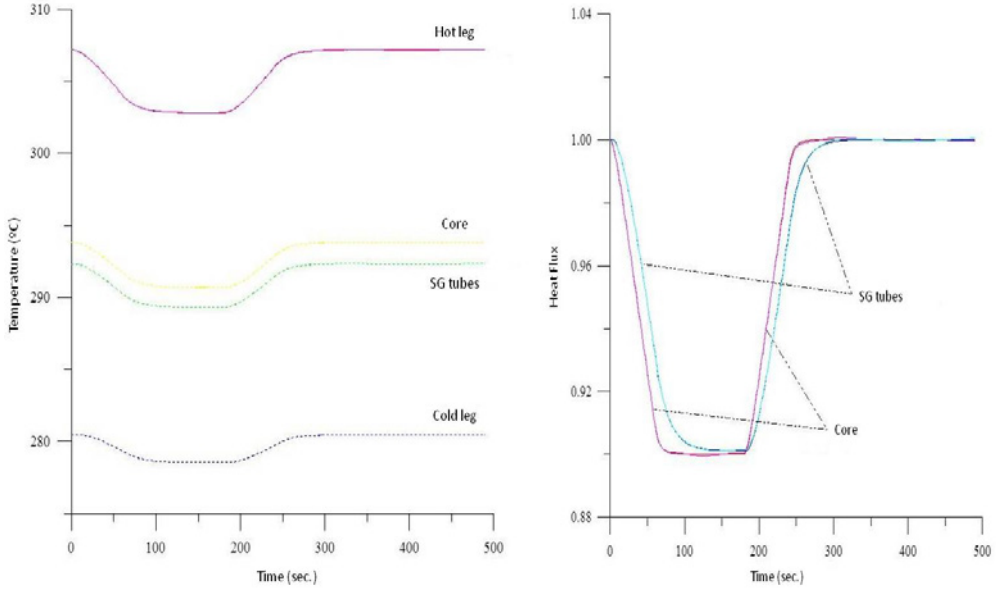


Figure 13(a). Exact and 15th-order Laguerre expansion models time response of the system to a perturbation in the core power.

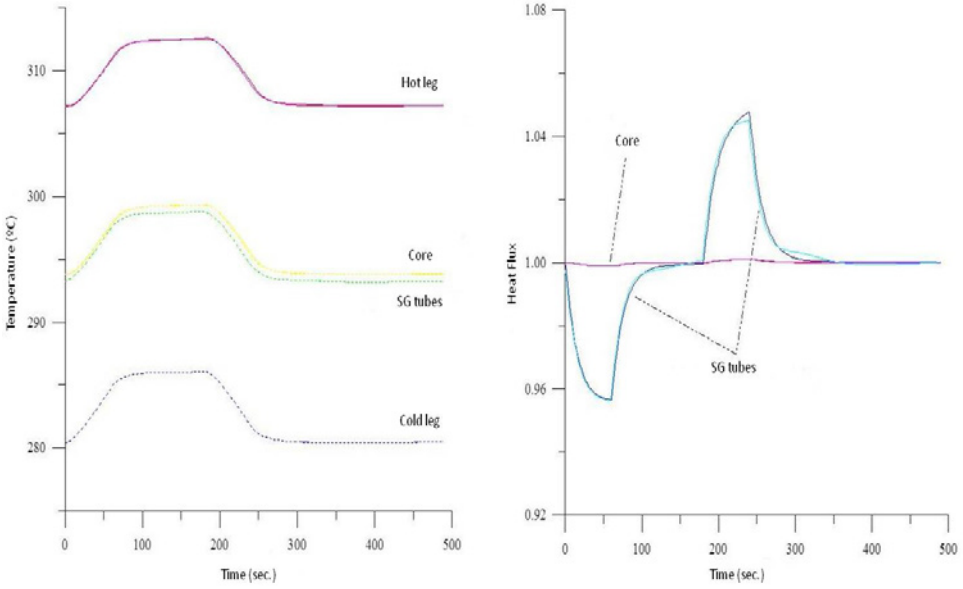


Figure 13(b). Exact and 15th order Laguerre expansion models time response of the system to a perturbation in the steam temperature.

5.1 Example of system identification.

As stated previously in Section 4, the coefficients of the proposed expansion can be determined from experimental data of the response of the system, according to relationships (10) and (11). The identification method is illuistrated in the Nuclear Power Plant model for the characterization of some of the significant variables of thermohydraulic processes (heat transmission and enthalpy transport) involved in the core and the steam generator (SG) of a PWR nuclear plant. As above, core power and steam temperature in the secondary side of the SG are used as boundary conditions of the model (i.e., input variables to the model).

Input variables functions that are used to identify the model are as above a load reduction 100% to 90% in the nuclear power and a smooth increase in the steam temperature of 6° C. The Laguerre coefficients of the transfer functions for the hot and cold temperatures, and average temperature in the primary circuit, core and SG tubes heat flows, are identified from the plant response obtained from the simulation of these transients with a more elaborated non-linear model.

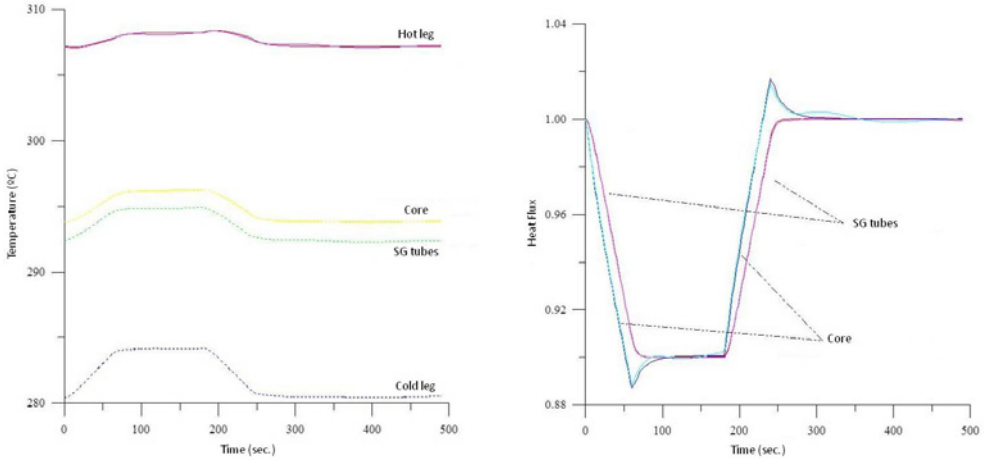


Figure 14. Exact and 15th order Laguerre expansion models time response of the system to simultaneous perturbations in the core power and in the steam temperature.

Figures 13 show the exact and approximate responses of the model for the same input variable, figure 13(a) for the core power transient and figure 13(b) for the steam temperature transient. In order to validate and verify the predictive capability of the model, two different transients are calculated. Figure 14 presents the results of the exact and approximate model when above input variables functions are simultaneous, while figures 15 are the exact and approximate time responses of the system to periodic variations in the nuclear power (figure 15a) and in the steam temperature (figure 15b). These results show the high degree of accuracy of the Laguerre expansion model.

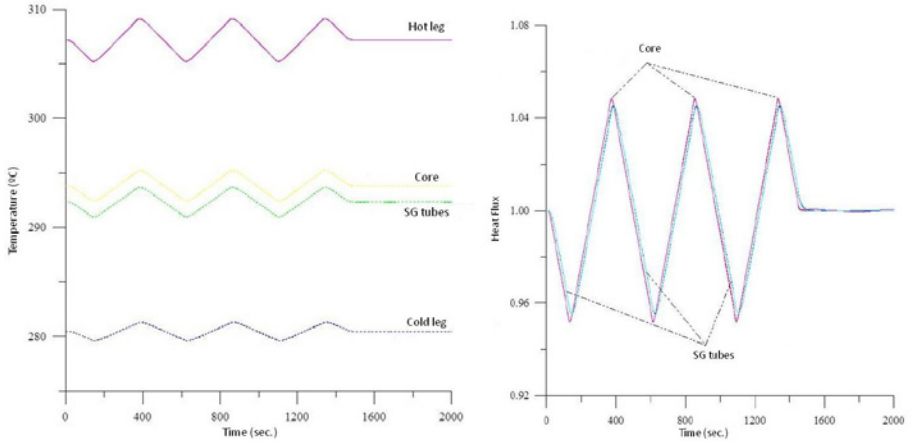


Figure 15(a). Exact and 15th-order Laguerre expansion models time response of the system to a sawtooth perturbation in the core power.

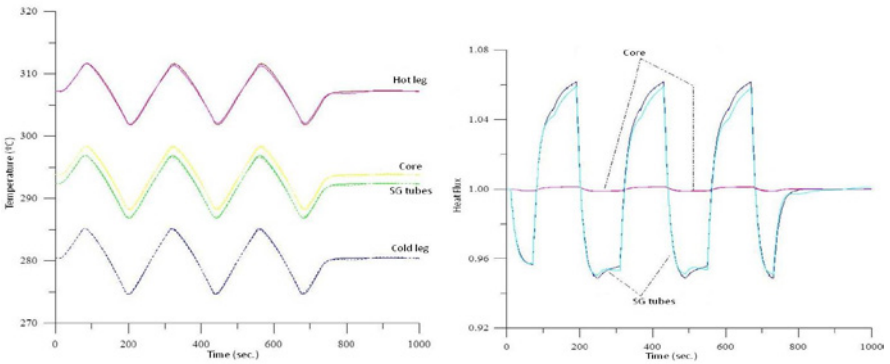


Figure 15(b). Exact and 15th-order Laguerre expansion models time response of the system to a sawtooth perturbation in the steam temperature.

VI. SUMMARY AND CONCLUSIONS.

This paper is part of a research project trying some new risk assessment techniques in the context of protection engineering (see [1], [2]). These require the use of large quantity of simulation runs of many accidental sequences of a large scale system model (such as a NPP), reason why fast and accurate dynamic simulation becomes compulsory. To accelerate the runs, a precomputed set of Laguerre coefficients of collapsed transfer functions is shown of interest. In another paper of the project, we demonstrate how to adapt these basic techniques to large scale piecewise linear systems by using transmission functions theory.

The paper has presented a feasible strategy for the simulation of large networks of linear subsystems (Large Scale Linear Systems, LSLS), including SISO and/or MIMO parts, and lumped or distributed (rational or non-rational transfer functions) linear elements, based on the use of the following ingredients ([14]):

1. A Modular and Structured modeling in terms of block diagrams.
2. The use of the collapsed description of each linear input/output pair transfer matrix, obtained by the application of the Mason's rule to the block diagram.
3. The Convolution Integral as form of treatment and definition of a linear system of any size or type.
4. The use of Laguerre Expansions as an efficient way to propose a numerical algorithm of calculation of convolutions.

The strategy has been presented through some examples of NPP transients, that demonstrate its feasibility.

REFERENCES

[1] Izquierdo J.M., Galushin S.E., Sánchez M., "Transmission Functions and its application to the analysis of time uncertainties in Protection Engineering", Process Safety and Environmental Protection (2013), http://dx.doi.org/10.1016/j.psep.2013.07.004.

[2] Izquierdo J.M., París C., Sánchez M., "The Theory of Transmission Functions and its Application to Protection Engineering", submitted to publication at the International Refereed Journal of Scientific Research in Engineering (IRJSRE) (December 2016)

[3] Muñoz Cobos J.L and Verdú G., Application of Hopf Bifurcation Theory and variational methods to the study of limit cycles in Boiling Water Reactors. Annals of Nuclear Energy Vol 18 N0 5 pp 269-302, 1991.

[4] March Leuba J. et al., Nonlinear dynamics and stability of Boiling Water Reactors. Part1 Qualitative Analysis Nucl. Sci. Eng. 93, 111-123 (1986) Part2 Quantitative Analysis Nucl. Sci. Eng. 93, 124-136 (1986) and several recent symposiums on the BWR stability like International Workshop on BWR Stability Oct 17-19, 1990 Brookhaven National Laboratory (Long Island, N.Y. USA).

[5] S.E. Salcudean, Pierre R. Belanger, Senior member, IEEE, Carl a. Goresky, and Colin P. Rose.The Use of Laguerre Functions for Parameter Identification in a Distributed Biological System. IEEE transactions on biomedical engineering, Volume-28, no. 11, November

[6] Astrom K. J. Computer Controlled Systems. Theory and Design (1988). Prentice Hall, Inc.

[7] R.E.Bellman and R.E.Kalaba Editors. Modern analytic and computational methods in Science and Mathematics. Applications of Numerical Inversion of the Laplace Transform American Elsevier Publishing company, Inc. New York 1966.

[8] BO Wahlberg, Member IEEE. System Identification Using Laguerre Models. IEEE Transactions on Automatic control, vol. 36, no. 5, May 1991.

[9] Guido Maione Combining Loop-shaping and Laguerre Development for Rational Approximation of a Noninteger Order, Integral, Analog Controller, Dipartimento di Ingegneria dell'Ambiente e per lo Sviluppo Sostenibile, Politecnico di Bari Viale del Turismo, 8 - 74100 Taranto – Italy.

[10] R. Malti, D. Maquin and J. Ragot, "Some results on the convergence of transfer function expansion on Laguerre series," Control Conference (ECC), 1999 European, Karlsruhe, 1999, pp. 4649-4655.

[11] S.J.Mason Feedback Theory: Further properties of signal flow graphs, Proc. IRE. 44, 7, July 1956 pp 920-926.

[12] Schwerg N., "Symbolical Analysis of RF-Network Problems using Mason's Rule", RadioEngeneering, Vol. 21, No. 3, September 2012.

[13] Hongyu Lu, Chongguang Wu, Shanglian Bao, "An Improved Algorithm based on Shannon-Happ Formula for Calculating Transfer Function from Signal Flow Graph and Its Visualization", arXiv e-print (arXiv:0906.5561).

[14] J.Hortal et al. COLAPSO-TRETA. A compact package for stability analysis and simulation of large integrated systems. Application to Nuclear Safety, International Forum "Mathematical Modelling and Computer Simulation of processes in Energy Systems" (Sarajevo, March 20-24, 1989).
[15] J.M. Izquierdo et al., SCAIS (Simulation Code System for Integrated Safety Assessment): Current status and applications, ESREL 2008 and 17th SRA Europe. Valencia (Spain), September, 2008.
[16] Sánchez Perea M., "Análisis y Simulación de Redes de Subsistemas Lineales mediante Desarrollos en Serie de Funciones de Laguerre", Tesis Doctoral, Escuela Técnica Superior de Ingenieros Industriales de ICAI, Universidad Pontificia Comillas, 1996.
[17] S Abate J, Choudhury G.L, Whitt W. On the Laguerre method for numerically inverting Laplace transforms. INFORMS Journal on Computing. 1996:413-27.
[18] Douglas G. Jones, Jose C. Principe Optimal Parameter Estimation of the Laguerre Filter via the Complex Performance Function, 1058-6393/97$ 10.00 0 1997 IEEE, Computational Neuroengineering Laboratory, CSE 447, University of Florida, Gainesville, FL 3261 1.
[19] L. Wang and W. R. Cluett, 0018-9286/94$04.00 0 1994 IEEE, Optimal Choice of Time-scaling Factor for Linear System Approximations Using Laguerre Models.
[20] Akcasu, Lellouche and Shotkin. Mathematical Methods in Nuclear Reactor Dynamics. Academic Press 1971.
[21] NENE 2015 Proceedings of the 24th International Conference "Nuclear Energy for New Europe".

Appendix A: Laguerre Coefficients Computation.

As far as this work is related, we have called "$\lambda\mu$-Laguerre coefficients" of the transfer function $K(s)$, denoted by and $\left\{k_n^{\lambda\mu}\right\}$, the projections with regard to the selected Laguerre functions, of the convolution kernel:

$$F(s) \equiv \frac{K(\lambda s-\mu)}{s} \tag{A.1}$$

These can be calculated, in a general way, as follows:

$$\begin{aligned} k_n^{\lambda\mu} &= \left\langle f(t), L_n(t)\right\rangle = \int_0^\infty e^{-t} L_n(t)dt = \pounds\left[f(t)L_n(t)\right]\Big|_{s=1} = \\ &= L_n\left(\frac{-d}{ds}\right)F(s)\Big|_{s=1} = L_n\left(\frac{-d}{ds}\right)\frac{K(\lambda s-\mu)}{s}\Big|_{s=1} \end{aligned} \tag{A.2}$$

where the following notation has been used:

$$L_n\left(\frac{-d}{ds}\right) \equiv \sum_{m=0}^{n}\binom{n}{m}\frac{1}{m!}\frac{d^m}{ds^m} \tag{A.3}$$

By using Parseval's Theorem, $\left\{k_n^{\lambda\mu}\right\}$ can also be obtained by

$$k_n^{\lambda\mu} = \frac{1}{2\pi j}\int_\Gamma F(s)L_n(s)ds = \frac{1}{2\pi j}\int_\Gamma \frac{K(\lambda s-\mu)}{s}\left(\frac{s}{s-1}\right)^n\frac{ds}{1-s} \tag{A.4}$$

with $L_n(s)$ the Laplace transform of $L_n(t)$.

If $K(s)$ is rational, corresponding to lumped parameters linear systems, its $\lambda\mu$-Laguerre coefficients can be calculated as the solution of the difference equation obtained from:

$$K_G^{\lambda\mu}(q) \equiv \sum_{n=0}^{\infty} k_n^{\lambda\mu} q^{-n} = \left[K(s)\right]_{s=-\mu+\frac{\lambda}{1-q^{-1}}} \tag{A.5}$$

Finally, it is possible to find expressions to compute the $\lambda\mu$-Laguerre coefficients knowing the frequency response of the system (i.e., $K(jw)$):

$$k_n^{\lambda\mu} = \frac{1}{2\pi}\int_{-\infty}^{\infty} K(j\omega)\left(\frac{j\omega+\mu}{j\omega+\mu-\lambda}\right)^n \frac{\lambda}{(j\omega+\mu)(\lambda-\mu-j\omega)}d\omega \tag{A.6}$$

Appendix B: Recursion Formulae for the Components of the Convolution Algorithm.

Functions defined in Section 4 have several recursion formulae useful for their computation. The efficiency of the Laguerre approach is partly due to these. For instance functions $\alpha_n(z)$ and $\phi_n(z)$ defined by (18) and (19) satisfy the following recursion relationships, where some other functions and $\mu' = \mu/\lambda$ are defined for convenience.

1. $$\alpha_n(z) \equiv e^{-\mu' z}\left(L_n(z) - L_{n-1}(z)\right)$$
$$(n+1)\alpha_{n+1}(z) = n\alpha_n(z) - ze^{-\mu' z}L_n(z)$$
$$\alpha_0(z) = e^{-\mu' z} \qquad \text{(B.1)}$$

2. $$\gamma_n(z) \equiv \int_0^z e^{-\mu' x}L_n(x)dx$$
$$\gamma_{n+1}(z) = \frac{(\mu'-1)\gamma_n(z) - \alpha_n(z)}{\mu'}$$
$$\gamma_0(z) = \frac{1-e^{-\mu' z}}{\mu'} \qquad \text{(B.2)}$$

3. $$\eta_n(z) \equiv \int_0^z \gamma_n(x)dx$$
$$\eta_{n+1}(z) = \frac{(\mu'-1)\eta_n(z) - (\gamma_{n+1}(z) - \gamma_n(z))}{\mu'}$$
$$\eta_0(z) = \frac{1-\gamma_0(z)}{\mu'} \qquad \text{(B.3)}$$

4. $$\phi_n(z) \equiv \int_0^z e^{-\mu' x}\frac{d}{dx}L_n(x)dx$$
$$\phi_{n+1}(z) = \phi_n(z) - \gamma_n(z)$$
$$\phi_n(z) = 1 \qquad \text{(B.4)}$$

Notice that for the usual case $\mu' = \mu/\lambda = 1$, these relations are simplified.

Appendix C: Transfer Functions of the Linear Model of the Primary Circuit of PWR (fig. 3).

The model depicted in figure 3 represents the processes of generation, transport and transfer of thermal energy from the reactor to the secondary side of the tubes of a steam generator (SG) in a pressurized water reactor nuclear power plant. Details and description of each one of these processes follow.

a) The neutronic power generation. The multiplicative process in the core is represented by means of the classical direct point kinetics, currently in used in many transient and accident analyses of nuclear power stations:

$$Y(s) = \frac{Z_0(s)}{1 + |k_0| Z_0(s)}$$
$$Z_0(s) = \frac{1}{s\left(\frac{\ell}{\beta} + \sum_i \frac{a_i}{s+\lambda_i}\right)} \qquad \text{(C.1)}$$
$$r(t) = k(t)\left(1 + y(t)\right)$$
$$P(t) = P_0\left(1 + y(t)\right)$$

that illustrates the typical nonlinear feedback of the multiplicative effect of the chain reaction. The following classical parameters and variables have been defined:

- prompt neutron generation time, ℓ;
- proportion of delayed neutrons, β;

- constant time generation and proportion of each group's deferred, $1/\lambda_i$,a_i ; and
- initial reactivity and core (neutronic) power, k_0 ,P_0 .

This description [C.1] includes the feedback inherent due to the Doppler and the moderator (coolant) temperature effects, being represented by two feedback coefficients, namely the Doppler only power coefficient and isothermal coefficient:

$$\begin{aligned} k_{Dopp}(t) &= \alpha_D \phi_f(t) \\ k_{mod}(t) &= \alpha_{iso} T_{avg}(t) \end{aligned} \tag{C.2}$$

providing with the inherent stability to the multiplicative process.

b) Heat transfer to the fuel process, through the fuel rods. The heat generated in the fuel is transferred to the coolant by conduction and convection processes through the fuel pellets, and through the gap and the cladding of the rods. The model assumes that:

- Full radial heat conduction in the pellet, gap and cladding, neglecting any axial heat conduction.
- Parameters involved in the heat balance (i.e., Fourier law of heat transfer), i.e., densities, heat capacities, heat transmission constants of fuel pellets, gap and cladding are functions of the axial variable but constant in time.
- Axial power distribution in the core remains fixed in time.

Then, the heat flux in the rod cladding (i.e., into the coolant) as a function of axial variable z is given by:

$$\phi'(s,z) = A_\phi(s)H(z)P(s) + A_T(s)T_{cool}(s,z) \tag{C.3}$$

where, $P(t)$ and $H(z)$ are namely, the total fuel power generated in the core and its axial distribution, $T_{cool}(s,z)$ is the coolant temperature for each axial dimension z, that are moduled through the following transfer functions:

$$\begin{aligned} A_\phi(s) &= \frac{1}{R_{vc}R_{fv}C_fC_vs^2 + (R_{vf}C_f + R_{vc}C_v + R_{vc}C_f)s + 1} \\ A_T(s) &= A_\phi(s)\frac{1 + R_{vf}C_fs}{R_{vc}} - \frac{1}{R_{vc}} \end{aligned} \tag{C.4}$$

and:

- $R_{fv} \equiv \left(\frac{1}{4\pi k_f} + \frac{1}{2\pi r_f h_{gap}}\right)$: the equivalent thermal resistance between the fuel and the gap (per unit of length);
- $R_{vc} \equiv \frac{1}{2\pi r_v h_{coolant}}$: the equivalent thermal resistance between the rod cladding and the coolant (per unit of length);
- $C_f \equiv \pi r_f^2 c_{pf}\rho_f$: heat capacity of the fuel (per unit of length);
- $C_v \equiv 2\pi r_v \Delta r_v c_{pv}\rho_v$: heat capacity of the rod cladding (per unit of length);
- k_f : thermal conductivity of the fuel;
- r_f : radius of the fuel pellet;
- h_{gap} : heat transfer coefficient of the gap;
- r_v : radius (average) of the rod caldding;
- Δr_v : cladding thickness; and
- $h_{coolant}$: heat transfer (convective) coefficient of the cladding-coolant interface.

c) Coolant transport through the core region of the reactor vessel. Heat transmitted to the coolant is transmitted by convection through the primary circuit. These processes are described by the classical balance equations for the mass, enthalpy and momentum, and considering the following additional assumptions:

- The coolant flow is constant in time.

- The properties of the fluid, in particular density and heat capacity, are not time-dependent.

With these, the output and average coolant temperatures in the core would be given by:

$$T_{hot}(s) = e^{-(\frac{s}{v}-\frac{A_T(s)}{Gc_p})L} T_{cold}(s) + P(s)A_\phi(s)\int_0^L e^{-(\frac{s}{v}s-\frac{A_T(s)}{Gc_p})(L-z)} \frac{H(z)}{Gc_p} dz$$

$$T_{avg}(s) = \frac{\left(1-e^{-(\frac{s}{v}-\frac{A_T(s)}{Gc_p})L}\right)}{\left(\frac{s}{v}-\frac{A_T(s)}{Gc_p}\right)L} T_{cold}(s) + P(s)\frac{A_\phi(s)}{L}\int_0^L dz\int_0^z e^{-(\frac{s}{v}s-\frac{A_T(s)}{Gc_p})(z-z')} \frac{H(z')}{Gc_p} dz' \quad \text{(C.5)}$$

Where the following parameters (i.e., constants) and functions have been considered:

- v : coolant velocity;
- G : coolant mass flow;
- c_p : coolant heat capacity;
- L : core length; and
- $T_{cold}(t)$: core inlet (cold) coolant temperature.

Transfer functions $F_T(s)$, $F_\phi(s)$, $F_{Tm}(s)$, $F_{\phi m}(s)$ of the block diagram of figure 3 are defined then as:

$$\begin{aligned} T_{hot}(s) &\equiv F_T(s)T_{cold}(s) + F_\phi(s)P(s) \\ T_{avg}(s) &\equiv F_{Tm}(s)T_{cold}(s) + F_{\phi m}(s)P(s) \end{aligned} \quad \text{(C.6)}$$

d) Coolant transport in plenums and non-heated primary pipes. The hypothesis of constant coolant velocity allows to represent these non-heated sections, using typical pure transport delay:

$$T_{out}(s) = e^{-\tau s}T_{in}(s) \quad \text{(C.7)}$$

where $\tau = L/v$, with L being the length of the corresponding pipe and v the coolant velocity.

e) Heat transfer and enthalpy transport through the steam generator tubes. With assumptions similar to those used in the core (i.e., constant flow, fluid and materials properties constant in time), and considering additionally the existence of a global heat transfer coefficient in the SG tubes, i.e.,

$$Q_{SG}(s) = UA\left(T_{avg}(s) - T_s(s)\right) \quad \text{(C.8)}$$

the outlet temperatures from the tubes ($T_{cold}(t)$), can be desreibed by:

$$T_{cold}(s) = e^{-(\tau s+u)}T_{hot}(s) + \frac{\left(1-e^{-(\tau s+u)}\right)}{\tau s+u} uT_S(s) \quad \text{(C.9)}$$

where:

- $u \equiv UA/Gc_p$;
- UA: global heat transfer coefficient in the SG tubes;
- G : coolant mass flow;
- c_p : coolant heat capacity;
- $\tau = L/v$: transport delay in the SG tubes;
- $T_{hot}(t)$: SG tubes inlet (hot) coolant temperatura; and.
- $T_s(t)$: secondary side steam temperature.

Finally, the average temperature in the tubes is described by:

$$T_{avg}(s) = \frac{1}{\tau s+u}\left(T_{hot}(s) - T_{cold}(s) + uT_S(s)\right) \quad \text{(C.10)}$$